<u>PHY102: GENERAL PHYSICS II</u>
<u>Outline: Gauss Law for Dielectric Materials, Magnetism: Magnetic field, Magnetic force on</u>
<u>a current carrying conductor.</u>
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<u>Gauss Law for Dielectric Materials</u>
By electrostatics, the Gauss law is given by
$$\nabla . E = \frac{\rho_f}{\epsilon_0} \qquad (1)$$
where ρ_f is the charge density due to free charges. Equation (1) is applicable in vacuum and it may
be reconsidered for dielectric media.
Gauss laws for dielectric materials have two forms:
1. Integral form
2. Differential form

<u>Integral form of Gauss law for dielectric materials</u>
The electric flux passing through the closed surface is given by
$$\oint E . ds = \frac{q}{\epsilon_0} \qquad (2)$$
In the presence of dielectric, the electric flux passing through the closed surface is given by
$$\oint E . ds = \frac{q}{k\epsilon_0} = \frac{q}{\epsilon} \qquad (3)$$
$$\oint (\epsilon E) . ds = \oint D . ds = q$$
where q enclosed by the Gaussian surface is a free charge only and $D = \epsilon E = k\epsilon_0 E$ is the
displacement vector.
Hence, the $\oint D . ds = q$ states that the surface integral of displacement vector 'D' over a closed
surface equals the free charge enclosed within the surface.

<u>Differential form of Gauss Law for Dielectric Materials</u>
For a dielectric material, the differential form of Gauss law state that the divergence of electric
displacement D equals the charge density due to free charges (ρ_f). i.e.
$$\nabla . D = \rho_f \qquad (4)$$
where $D = \epsilon_0 E + P$ is the electric displacement having the same dimension as P (dipole moment
per unit volume).

<u>Magnetism</u>
The word magnet was coined after the name of an ancient town called **Magnesia** in Asia, where
the first magnetic effect were observed. The natural magnets are capable of attracting to themselves
unmagnetised iron. The effect of this attraction is stronger at the poles (ends) than at the centres.
The word magnetism is the study of **magnetic effects** as the **magnetic forces** interact with the
moving charges. From the experimental observations, it was concluded that:
- Like poles repel each other while unlike poles attract each other.
- Magnetic effect is stronger at the poles than at the centres.

- Poles seem to occur in equal and opposite pairs, say north and south poles.
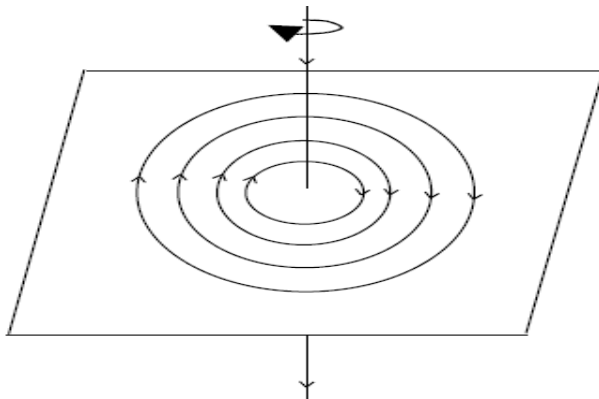
Magnetic Field

The space or region around a magnet especially around the poles where magnetic force is experience is referred to as magnetic field. The magnetic field exert a force on a moving charges such as conductors carrying currents. These magnetic forces find their applications in the operation of electric motors, moving-coil galvanometer and many other devices. The direction of a magnetic field at a point is taken as the direction of the force that acts on a north magnetic pole there. A magnetic field can be represented by magnetic field lines drawn so that:

(i) The line (or the tangent to it if is curved) gives the direction of the field at that point, and

(ii) The number of lines per unit cross-section area is an indication of the "strength" of the field.

A neutral point is a place where two magnetic fields are equal and opposite and the resultant force is zero.

Field Due to Currents

Figure 1

A conductor carrying an electric current is surrounded by a magnetic field. The lines due to a straight wire are circles, concentric with the wire as shown in fig. 1. The right-hand screw rule is a useful aid for predicting the direction of the field, knowing the direction of the current. It states that:

If a right-handed screw moves forward in the direction of the current (conventional), then the direction of rotation of the screw gives the direction of the lines.
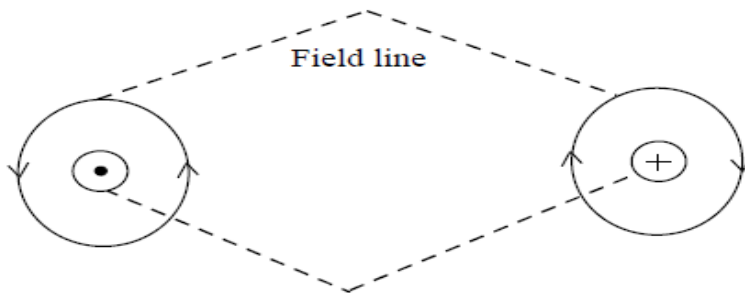
Figure 2 (a)   **Figure 2 (b)**

Figure 2 a and b illustrate the rule. In (a) the current is flowing out of the paper and the dot in the centre of the wire is the point of an approaching arrow; in (b) the current flowing into the paper and the cross is the tail of a receding arrow.

Some properties of Magnetic field

- The earth itself behaves like a permanent magnet
- At the magnetic poles the magnetic field is vertical
- The direction of magnetic field at a point is taken as the direction of magnetic force that acts on magnetic north pole.
- Magnetic field is a vector field, which is associated with each point in space.
- Magnetic field is denoted by $\bar{B}$.
- At any position, the direction of magnetic field is that in which North Pole of a compass needle tends to point.

Magnetic field lines

A magnetic field can be represented by magnetic field lines which are characterized as follows:

(i) Magnetic field line at any point is a tangent to the field $\bar{B}$. at that point.

(ii) The number of lines per unit cross-section area is an indication of the magnetic field strength. The closer together the field lines, the larger or stronger the field magnitudes and the farther apart the field lines, the smaller the field magnitudes

(iii) The arrow head of the field lines point away from the north poles and toward the south poles.

(iv) The arrow head of the field also represent the direction of the field.

(v) Because the direction of magnetic field at each point is unique, so the field lines never intersect.

(v) In a uniform field, the field lines are approximately straight, parallel and equally spaced.

Force on charge moving in a magnetic field

The magnetic force on a charge q moving with the velocity v through the magnetic field B is given as

$$\bar{F} = q\bar{v} \times \bar{B} \tag{5}$$
$$\bar{v} \times \bar{B} = vB\sin\theta \tag{6}$$
$$F = qvB\sin\theta \tag{7}$$

where θ is the angle between v and B. Equation (7) implies that the direction of F is always perpendicular to the plane containing $\bar{v}$ and $\bar{B}$. The force is equal to zero if the charge is zero. Also, the magnitude of the force is zero if v and B are either parallel ($\theta = 0°$) or antiparallel ($\theta = 180°$) and the force is maximum when $\bar{v}$ and $\bar{B}$ are perpendicular to each other ($\theta = 90°$).

Note the following:

i. If q is positive, the direction of $\bar{F}$ is the same direction of $\bar{v} \times \bar{B}$.

ii. If q is negative, the direction of $\bar{F}$ is opposite to the direction of $\bar{v} \times \bar{B}$.

iii. The S.I unit of B is Tesla (T) or Weber per square metre (Wb/m^2)

$$1T = \frac{N}{C.m/s} = \frac{N.s}{C.m} = \frac{N}{A.m}, \text{ where Ampere } A = C/s$$

iv. Gaussmeter is an instrument for measuring magnetic field. The magnetic field of the earth is of the order of $10^{-4}T = 1\ G$ i.e. $1\ Gausss\ G = 10^{-4}Tesla\ T$.

v. The largest magnetic field that can be produced in the laboratory is $45\ T$.

vi. When a charges particle moves through a region of space, where both electric and magnetic fields are present, both fields exerts forces on the particle and the total force is the vector sum of the electric and magnetic forces given by

$$\bar{F} = q(\bar{E} + \bar{v} \times \bar{B}) \tag{8}$$

Equation (8) is also known as Lorentz force.

Example 1: A beam of protons (q=1.6 x 10^{-19} C) moves at 3.0 x 10^5 m/s through a uniform magnetic field with magnitude 2.0 T along positive z-axis. (i) determine the velocity of each proton lies in the xz plane at an angle of 30° to the +z axis. (ii) find the force on a proton.

Solution

$F = qvBsin\theta$ = 1.6 x 10^{-19} C x 3.0 x 10^5 m/s x 2.0 T x sin 30° = 4.8 x 10^{-14} N

Note that if the beam consists of electrons, the charge is negative and the direction of the force is along positive y-axis but the magnitude of the magnetic force is the same as 4.8 x 10^{-14} N.

Tutorial 1: An electron in a TV camera tube is moving at 7.60 x 10^6 m/s in a magnetic field of strength 83.0 mT. At one point, the electron has an acceleration of magnitude 4.70 x 10^{14} mls^2. What is the angle between the velocity and the field? Take $m_e = 9.11 \times 10^{-31}kg, q = 1.6 \times 10^{-19}C$. Hint: use $F = m_e a = qvBsin\theta$

Tutorial 2: A proton travels at 23° with respect to the direction of magnetic field of strength 26 mT experience a force of 6.50 x 10^{-17} N. Calculate (a) the proton`s speed (b) its kinetic energy in electron volt (eV).

Magnetic Flux Φ_B

This is the amount of magnetic field B that passes through a loop that can be defined in a similar way as electric flux Φ_E

$$\Phi_E = \int \bar{E}.d\bar{A} \tag{9}$$

Suppose a loop enclosing an area A is placed in a magnetic field $\bar{B}$, then the magnetic flux Φ_B through the loop is given by

$$\Phi_B = \int \bar{B}.d\bar{A} \tag{10}$$

By definition and using Gaussian surface, the area vector $d\bar{A}$ at any point is perpendicular to the surface and has magnitude dA. By applying dot product the $\int \bar{B}.d\bar{A}$ in equation (10) becomes

$$\Phi_B = \int BdAcos\theta = \int Bcos\theta dA \tag{11}$$

where θ is the angle between area A and magnetic field B.

Suppose that the loop lies in a plane where magnetic field $\bar{B}$ is perpendicular to the plane of the loop, then $cos\theta = 1$ and the magnetic flux Φ_B through the loop becomes

$$\Phi_B = \int BdA = BA \tag{12}$$

The magnetic flux Φ_B is a scalar quantity measured in Tesla metre square (Tm^2) or weber (Wb). Note that 1 Wb = 1 Tm^2 =1 Nm/A

The Gauss`s law for magnetism states that the total magnetic flux through a closed surface is always zero.

i.e.
$$\oint \bar{B}.d\bar{A} = 0 \tag{13}$$

If element of area dA is at right angle to the field lines, then

$$B = \frac{d\Phi_B}{dA_{perpendicular}}$$ (14)

This implies that the magnitude of magnetic field is equal to flux per unit area across an area at right angles to the magnetic field. This is why magnetic field B is sometimes called magnetic flux density.

Motion of charge in a uniform magnetic field

For a charge q travelling with velocity v in a uniform magnetic field B perpendicular to the direction of the velocity, the charge exhibits uniform circular motion with radius R and the centripetal acceleration is given by

$$a = \frac{v^2}{R}$$ (15)

By 2nd law of motion

$$F = ma = \frac{mv^2}{R}$$ (16)

The magnetic force F acting on the particle counterbalances the force of motion such that

$$F = qvB = \frac{mv^2}{R} \text{ since } \theta = 90°$$ (17)

where m is the mass of the particle and from equation (17) we can write the radius of the circular orbit in a magnetic field as

$$R = \frac{mv}{qB} = \frac{P}{qB}$$ (18)

where P is the momentum of the particle. If q is negative, the particle moves clockwisely around the orbit and conversely. Equations (17) and (18) show that for a certain velocity V in a fixed magnetic field strength B, a charge q with mass m has a fixed radius of orbit. The angular velocity ω of the particle can be found from equation (17) as

$$\omega = \frac{v}{R} = \frac{qB}{m}$$ (19)

The number of revolutions per unit time is given by

$$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$ (20)

Equation (20) shows that the frequency f is independent of radius R of the path and the velocity of the particle but depend on the charge to mass ratio (q/m). The frequency f is now called the Cyclotron frequency. It is the frequency at which the charged particles circulate in a cyclotron particle accelerator. The period of the particle`s orbit is obtained from equation (20) as

$$T = \frac{1}{f} = \frac{2\pi m}{qB}$$ (21)

We should take note that no work is performed by magnetic force on a moving charge. i.e. the kinetic energy of the particle is not changing by the magnetic field.

Tutorial 3

Estimate the cyclotron frequency if the magnetic field strength is 2.0 x10^{-4} T.

Tutorial 4

A magnetron in a microwave oven emits electromagnetic waves with frequency 2540 MHz. What magnetic field strength is required for electrons to move in circular paths with this frequency?

Ans B=0.0877T

Magnetic force on a current-carrying conductor

When a current-carrying conductor lies in a magnetic field, magnetic force are exerted on the moving charges within the conductor. These forces are transmitted to the material of the conductor, and the conductor as a whole experienced a force distributed along its length. The electric motor and the moving coil galvanometer both depend on their operation on the magnetic force on conductor – carrying currents.

The force on a current-carrying conductor

(i) always perpendicular to the plane containing the conductor and the direction of the field in which it is placed and

(ii) greatest when the conductor is at right angles to the field.
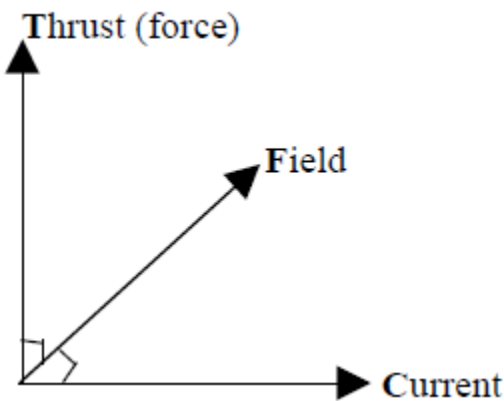

Figure 1: Fleming's left-hand (or motor) rule

The facts about the relative directions of current, field and force are summarized by Fleming's left-hand rule which states that:

If the thumb and first two fingers of the left-hand are held each at right angles to the other, with the first Finger pointing in the direction of the Field and the second finger in the direction of the Current, then the thumb predicts the direction of the Thrust or force. See figure 1.

Factors Affecting the Force

The force F on a wire lying at right angles to a magnetic field is directly proportion to the current I in the wire and to the length l of the wire in the field. It also depends on the magnetic field.

The magnetic force that acts on a length l of a straight wire carrying a current I when immersed in a magnetic field B that is perpendicular to the wire is given by

$$F = BIl \qquad (22)$$

And the generalized form is given by

$$\bar{F} = I\bar{l} \times \bar{B} \qquad (23)$$

where $\bar{l}$ is a length vector that has magnitude l and is directed along the wire segment in the direction of the current. So the magnitude of $\bar{B}$ is

$$F = BIl\sin\theta \qquad (24)$$

where θ is the angle between the direction of $\bar{F}$ and $\bar{B}$ and force F is perpendicular to both the conductor and the field. If a wire is not straight or the field is not uniform, we can imagine it (wire) broken up into small straight segments $d\bar{l}$, then we can apply equation (23) to each segment. The force F on the whole wire is the vector sum of all the forces $d\bar{F}$ in the segments $d\bar{l}$ that is made up of the whole wire.

$$d\bar{F} = Id\bar{l} \times \bar{B} \quad (\text{Magnetic force on each segment}) \qquad (25)$$

Torque on a current loop

The torque on a coil of wire carrying a current I that is immersed in a magnetic field B is given as
$$\tau = BIA \sin\theta \quad (\text{magnitude of torque on current loop}) \qquad (26)$$
where θ is the angle between the field and the normal, A is the area. If the coil has N- loops or turns, then the total torque is now given as
$$\tau = NBIA \sin\theta \qquad (27)$$

Magnetic Dipole Moment

The magnetic dipole moment $\bar{\mu}$ has the magnitude NIA. By relating this with the torque in magnetic field B, we have

$$\tau = \mu B \sin\theta \quad (\text{where } \mu = NIA) \qquad (28)$$

The unit of magnetic dipole moment is Am^2

In vector notation we can re-write equation (28) as

$$\tau = \bar{\mu} \times \bar{B} \qquad (29)$$

Example 2: What is the force exerted on a straight wire of length 3.5cm, carrying a current of 5A, and situated at right angles to a magnetic field of flux density 0.2T ?
Solution:
$F = BIl = 0.2 \times 3.5 \times 10^{-2} \times 5 = 0.035\ N$
Example 3: Find the torque on a galvanometer coil, 2cm square and containing 100 turns, when a current of 1mA passes through it. The radial field of the permanent magnet has a flux density of 0.2T.
$\tau = NBIA = 100 \times 0.2 \times 1 \times 10^{-3} \times 2 \times 10^4 = 8 \times 10^{-6} Nm$